Prabhu Classes

JEE-MAIN 2020

Time : 75 Min	Maths : Straight Line	Marks : 100

01) The equation to pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. Then what are the equations to its diagonals?
A) $4x + y = 13, 4y = x - 7$
B) $x + 4y = 13, y = 4x - 7$
C) $4x + y = 13, y = 4x - 7$
D) $y - 4x = 13, y + 4x = 7$

02) What is the area enclosed by $2|x| + 3|y| \le 6$?
A) 4 sq. units
B) 3 sq. units
C) 12 sq. units
D) 24 sq. units

03) The product of the perpendiculars drawn from the points $(\pm\sqrt{a^2 - b^2}, 0)$ on the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$, is
A) b^2
B) a^2
C) $a^2 + b^2$
D) $a^2 - b^2$

04) If p and p' be the distances of origin from the lines $x \sec\alpha + y \operatorname{cosec}\alpha = k$ and $x\cos\alpha - y\sin\alpha = k\cos 2\alpha$, then $4p^2 + p'^2 =$
A) $2k^2$
B) k^2
C) 2k
D) k

05) The value of λ for which the lines $3x + 4y = 5$, $5x + 4y = 4$ and $\lambda x + 4y = 6$ meet at a point is
A) 1
B) 2
C) 3
D) 4

06) If the lines $ax + y + 1 = 0, x + by + 1 = 0$ and $x + y + c = 0$ (a,b,c being distinct and different from 1) are concurrent, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$
A) 1
B) 0
C) $\frac{1}{a+b+c}$
D) None of these

07) If a variable line drawn through the point of intersection of straight lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $\frac{x}{\beta} + \frac{y}{\alpha} = 1$ meets the coordinate axes in A and B, then the locus of the mid point of AB is
A) $\alpha\beta(x + y) = 2xy(\alpha + \beta)$
B) $\alpha\beta(x + y) = xy(\alpha + \beta)$
C) $(\alpha + \beta)(x + y) = 2\alpha\beta xy$
D) None of these

08) If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is
A) Circle
B) Square
C) Straight line
D) Two intersecting lines

09) What is the locus of the foot of the perpendicular from the origin on each member of the family $(4a + 3)x - (a + 1)y - (2a + 1) = 0$?
A) $(2x - 1)^2 + (y + 1)^2 = 5$
B) $(2x - 1)^2 + 4(y + 1)^2 = 5$
C) $(2x + 1)^2 + 4(y - 1)^2 = 5$
D) $(2x - 1)^2 + 4(y - 1)^2 = 5$

10) What is the locus of the centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$, and $(1, 0)$, where t is a parameter?
A) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$
B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
C) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$
D) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

11) What is the distance of the origin from the line $(1 + \sqrt{3})y + (1 - \sqrt{3})x = 10$ along the line $y = \sqrt{3}x + k$?
A) $5\sqrt{2} + k$
B) $5/\sqrt{2}$
C) 10
D) 0

12) Calculate the number of points having both coordinates as integers, that lie in the interior of the triangle with vertices $(0, 0)$, $(0, 41)$ and $(41, 0)$

A) 901
B) 820
C) 861
D) 780

13) The line joining two points A(2,0), B(3,1) is rotated about A in anti-clockwise direction through an angle of 15^o. The equation of the line in the new position, is
A) $x-3\sqrt{y}-2=0$
B) $\sqrt{3}x-y-2\sqrt{3}=0$
C) $\sqrt{3}x+y-2\sqrt{3}=0$
D) $x+3\sqrt{y}-2=0$

14) If a pair of perpendicular straight lines drawn through the origin forms an isosceles triangle with the line $2x+3y=6$, then what is the area of the triangle so formed?
A) 36/13
B) 13/5
C) 12/17
D) 17/13

15) A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. What is its equation?
A) $x+y=7$
B) $3x+4y=25$
C) $4x+3y=24$
D) $3x-4y+7=0$

16) The lines $ax+by+c=0$, where $3a+2b+4c=0$ are concurrent at the point
A) (1/2,3/4)
B) (1,3)
C) (3,1)
D) (3/4,1/2)

17) The area of the parallelogram formed by the lines $y=mx, y=mx+1, y=nx$ and $y=nx+1$ equals
A) $\frac{2}{|m+n|}$
B) $\frac{1}{|m+n|}$
C) $\frac{1}{|m-n|}$
D) $\frac{|m+n|}{(m-n)^2}$

18) In what direction a line be drawn through the point (1,2) so that its points of intersection with the line $x+y=4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point
A) 75^o
B) 60^o
C) 45^o
D) 30^o

19) If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
A) Lie on an ellipse
B) Lie on a straight line
C) Lie on a circle
D) Are vertices of a triangle

20) A point starts moving from (1,2) and its projections on x and y-axes are moving with velocities of $3m/s$ and $2m/s$ respectively. Its locus is
A) $2y-3x+1=0$
B) $2x-3y+4=0$
C) $3x-2y+1=0$
D) $3y-2x+4=0$

21) A square of side a lies above the x -axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\pi/4\right)$ with the positive direction of x -axis. What is the equation of its diagonal not passing through the origin?
A) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$
B) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$
C) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$
D) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$

22) Through the point $P(\alpha,\beta)$, where $\alpha\beta>0$, the straight line $\frac{x}{a}+\frac{y}{b}=1$ is drawn so as to form with axes a triangle of area S. If $ab>0$, then find out least value of S.
A) $3\alpha\beta$
B) $2\alpha\beta$
C) $\alpha\beta$
D) None of these

23) A line L passes through the points (1,1) and (2,0) and another line L' passes through $\left(\frac{1}{2},0\right)$ and perpendicular to L. Then the area of the triangle formed by the lines L,L' and y-axis, is
A) 25/16
B) 25/8
C) 25/4
D) 15/8

24) A ray of light coming from the point (1,2) is reflected at a point A on the x-axis and then passes through the point (5,3). The coordinates of the point A are
A) $(5/13, 0)$

B) $(13/5, 0)$
C) $(-7, 0)$
D) None of these

25) If straight lines $ax + by + p = 0$ and $x\cos\alpha + y\sin\alpha - p = 0$ include an angle $\pi/4$ between them and meet the straight line $x\sin\alpha - y\cos\alpha = 0$ in the same point, then evaluate the value of $a^2 + b^2$.

Prabhu Classes

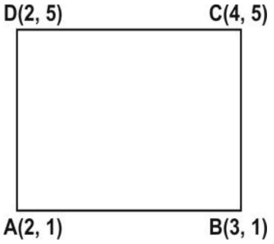
JEE-MAIN 2020

Time : 75 Min	Maths : Straight Line	Marks : 100

Hints and Solutions

01) Ans: **C)** $4x + y = 13, y = 4x - 7$

Sol: The side of parallelogram are
$x = 2, x = 3, y = 1, y = 5$.



The equation of diagonal AC is $\frac{y-1}{5-1} = \frac{x-2}{3-5}$

or $y = 4x - 7$ and the equation of diagonal BD is

$\frac{x-2}{3-2} = \frac{y-5}{1-5}$ or $4x + y = 13$

02) Ans: **C)** 12 sq. units

Sol: The given inequality is equivalent to the following system of inequalities.

$2x + 3y \le 6$, when $x \ge 0, y \ge 0$

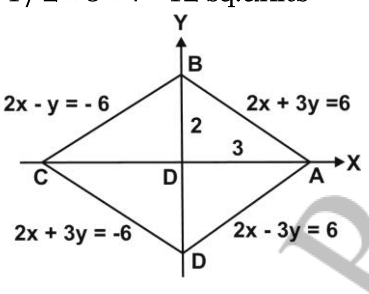
$2x - 3y \le 6$, when $x \ge 0, y \le 0$

$-2x + 3y \le 6$, when $x \le 0, y \ge 0$

$-2x - 3y \le 6$, when $x \le 0, y \le 0$

which represents a rhombus with sides
$2x \pm 3y = 6$ and $2x \pm 3y = -6$

The length of the diagonals are 6 units and 4 units respectively, along x and y-axes.
Therefore, the required area is
$1/2 \times 6 \times 4 = 12$ sq.units



03) Ans: **A)** b^2

Sol:

$$\left(\frac{b\sqrt{a^2-b^2}\cos\theta + 0 - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right)\left(\frac{-b\sqrt{a^2-b^2}\cos\theta - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right)$$

$$= \frac{-[b^2(a^2-b^2)\cos^2\theta - a^2b^2]}{(b^2\cos^2\theta + a^2\sin^2\theta)}$$

$$= \frac{b^2[a^2 - a^2\cos^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2[a^2\sin^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta} = b^2$$

04) Ans: **B)** k^2

Sol: $p = \left|\frac{-k}{\sqrt{\sec^2\alpha + \text{cosec}^2\alpha}}\right|, p' = \left|\frac{-k\cos 2\alpha}{\sqrt{\cos^2\alpha + \sin^2\alpha}}\right|$

$$\therefore 4p^2 + p' \frac{4k^2}{\sec^2\alpha + \text{cosec}^2\alpha} + \frac{k^2(\cos^2\alpha - \sin^2\alpha)^2}{1}$$

$$\Rightarrow = 4k^2\sin^2\alpha\cos^2\alpha + k^2(\cos^4\alpha + \sin^4\alpha)$$

$$-2k^2\cos^2\alpha\sin^2\alpha$$

$$\Rightarrow = k^2(\sin^2\alpha + \cos^2\alpha)^2 = k^2$$

05) Ans: **A)** 1

Sol: The lines are $3x + 4y = 5$, $5x + 4y = 4$, $\lambda x + 4y = 6$. These lines meet at a point if point of intersection of first two lines lies on third line.

From $3x + 4y = 5$ and $5x + 4y = 4$,

$x = \frac{-1}{2}$, $y = \frac{13}{8}$

This lies on $\lambda x + 4y = 6$, if $\lambda\left(-\frac{1}{2}\right) + 4\left(\frac{13}{8}\right) = 6$

$\Rightarrow \lambda = 1$

06) Ans: **A)** 1

Sol: If the lines are concurrent,

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

{Apply $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$}

$\Rightarrow a(b-1)(c-1) - (b-1)(1-a) - (c-1)(1-a) = 0$

$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

{Divide by $(1-a)(1-b)(1-c)$}

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

07) Ans: **A)** $\alpha\beta(x+y) = 2xy(\alpha+\beta)$

Sol: Equation of a line passing through intersection of straight lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $\frac{x}{\beta} + \frac{y}{\alpha} = 1$ is

$$\left(\frac{x}{\alpha} + \frac{y}{\beta} - 1\right) + \lambda\left(\frac{x}{\beta} + \frac{y}{\alpha} - 1\right) = 0$$

or $x\left(\frac{1}{\alpha} + \frac{\lambda}{\beta}\right) + y\left(\frac{1}{\beta} + \frac{\lambda}{\alpha}\right) - \lambda - 1 = 0$

It meets the axes at

$A\left(\dfrac{\lambda+1}{\frac{1}{\alpha}+\frac{\lambda}{\beta}},0\right)$ and $B\left(0,\dfrac{\lambda+1}{\frac{1}{\beta}+\frac{\lambda}{\alpha}}\right)$

Let (h, k) be the mid point of AB,

$h=\dfrac{1}{2}\cdot\dfrac{\lambda+1}{\frac{1}{\alpha}+\frac{\lambda}{\beta}}, k=\dfrac{1}{2}\cdot\dfrac{\lambda+1}{\frac{1}{\beta}+\frac{\lambda}{\alpha}}$

By eliminating λ from these two,

$2hk(\alpha+\beta)=\alpha\beta(h+k)$

$\therefore$ Locus of (h,k) is $2xy(\alpha+\beta)=\alpha\beta(x+y)$

08) Ans: **B)** Square

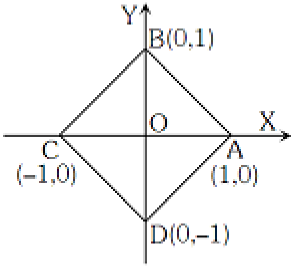
Sol: The required locus of point (x, y) is the curve $|x|+|y|=1$

If point lies in the first quadrant, $x>0, y>0$

and $|x|+|y|=1 \Rightarrow x+y=1$, which is straight line AB

If point (x, y) lies in second quadrant, $x<0$, $y>0$

and $|x|+|y|=1 \Rightarrow -x+y=1$



Same for third and fourth quadrant, the eq. are $-x-y=1$ and $x-y=1$.

$\therefore$ The required locus is the curve consisting of the sides of the square ABCD.

09) Ans: **D)** $(2x-1)^2+4(y-1)^2=5$

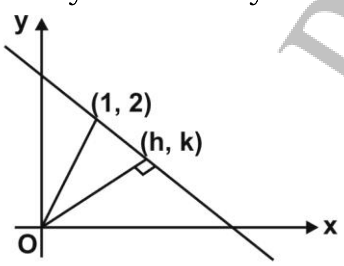
Sol: Given family of lines is

$(4a+3)x-(a+1)y-(2a+1)=0$

or $(3x-y-1)+a(4x-y-2)=0$

Family of lines passes through the fixed point P which is the intersection of

$3x-y=1$ and $4x-y=2$ Solving we get $P(1,2)$.



Now let (h,k) be the foot of perpendicular on each of the family.

$\therefore \dfrac{k}{h}\cdot\dfrac{k-2}{h-1}=-1 \quad \Rightarrow$ Locus is$(x-x)$ 1 $(y-y)$ 2

or $(2x-1)^2+4(y-1)^2=5$

10) Ans: **B)** $(3x-1)^2+(3y)^2=a^2+b^2$

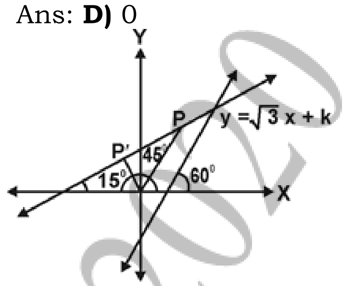
Sol: Let the centroid be (x, y)

$\therefore x=\dfrac{a\cos t+b\sin t+1}{3} \Rightarrow a\cos t+b\sin t=3x-1$

$y=\dfrac{a\sin t-b\cos t}{3} \Rightarrow a\sin t-b\sin t=3y$

Squaring and adding, we get $(3x-1)^2+(3y)^2$ $=a^2+b^2$

11) Ans: **D)** 0

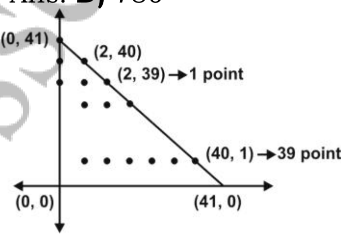


Sol:

The angle between both the lines is 45^0.

Therefore, $OP=OP'\sqrt{2}=\left(\dfrac{5}{\sqrt{2}}\right)\times\sqrt{2}=5$.

12) Ans: **D)** 780



Sol:

Thus, the number of points =1+2+3+...+39

$=\dfrac{39\times 40}{2} = 780$

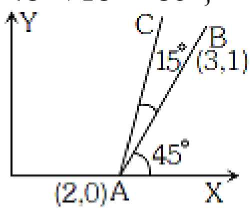
13) Ans: **B)** $\sqrt{3}x-y-2\sqrt{3}=0$

Sol: Slope of $AB=\dfrac{1}{1} \Rightarrow \tan\theta=m_1=1$ or $\theta=45^o$.

The slope of new line is

$\tan(45^o+15^o)=\tan 60^o=\sqrt{3}$

{$\because$ It is rotated anticlockwise so angle will be $45^o+15^o=60^o$}



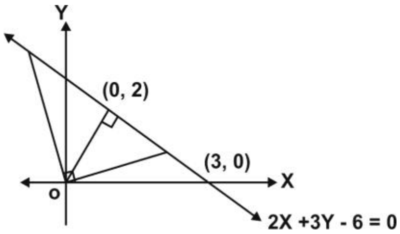
The equation is $y=\sqrt{3}x+c$, but it still passes through (2,0), $\therefore c=-2\sqrt{3}$.

The required equation is $y=\sqrt{3}x-2\sqrt{3}$

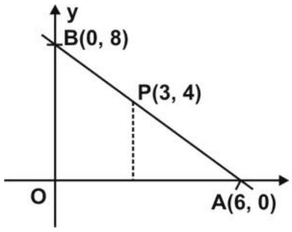
14) Ans: **A)** 36/13

Sol: The distance of $(0,0)$ from the line $2x+3y-6=0$ is $6/\sqrt{4+9}=6/\sqrt{13}$.

The area of the triangle is $\left(6/\sqrt{13}\right)^2 = 36/13$



15) Ans: **C)** $4x+3y=24$



Sol:
Since P is mid point of the line, intercept on axes are 6 and 8.

Therefore, the equation of line is $\frac{x}{6}+\frac{y}{8}=1$

or $4x+3y=24$

16) Ans: **D)** (3/4,1/2)
Sol: By dividing both sides of relation

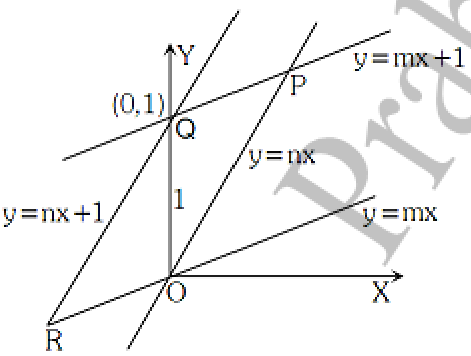
$3a+2b+4c=0$ by 4, $\frac{3}{4}a+\frac{1}{2}b+c=0$,

which shows for all values of a, b and c each member of set of lines $ax+by+c=0$ passes through point $\left(\frac{3}{4},\frac{1}{2}\right)$

17) Ans: **C)** $\frac{1}{|m-n|}$

Sol: By solving $y=nx$ and

$y=mx+1$, $P=\left(\frac{1}{n-m},\frac{n}{n-m}\right)$



$\therefore$ Area of parallelogram = 2×(area of ΔPOQ)

$=2\times\left|\frac{1}{2}\times OQ\times\frac{1}{n-m}\right|=\frac{1}{|n-m|}=\frac{1}{|m-n|}$.

18) Ans: **A)** 75^o
Sol: Let required line through the point (1,2) be inclined at angle θ to the axis of x,

its equation is $\frac{x-1}{\cos\theta}=\frac{y-2}{\sin\theta}=r$ ....(i)

where r is distance of any point (x, y) on the line from point (1,2).
The coordinates of any point on line (i) are $(1+r\cos\theta, 2+r\sin\theta)$.

If this point is at distance $\frac{\sqrt{6}}{3}$ form (1,2),then

$r=\frac{\sqrt{6}}{3}$.

$\therefore$ The point is, $\left(1+\frac{\sqrt{6}}{3}\cos\theta, 2+\frac{\sqrt{6}}{3}\sin\theta\right)$

But this point lies on line $x+y=4$.

$\Rightarrow \frac{\sqrt{6}}{3}(\cos\theta+\sin\theta)=1$ or $\sin\theta+\cos\theta=\frac{3}{\sqrt{6}}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta=\frac{\sqrt{3}}{2}$

{Dividing both sides by $\sqrt{2}$}

$\Rightarrow \sin(\theta+45^o)=\sin 60^o$ or $\sin 120^o$

$\Rightarrow \theta=15^o$ or 75^o

19) Ans: **B)** Lie on a straight line

Sol: By taking co-ordinates as $\left(\frac{x}{r},\frac{y}{r}\right)$; (x,y) and (xr,yr)

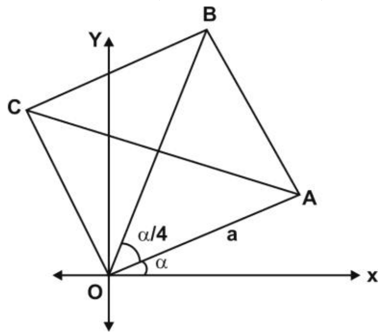
Above co-ordinates satisfy relation $y=mx$, so lie on a straight line.

20) Ans: **B)** $2x-3y+4=0$
Sol: After time 't' secs.
Abscissa, $x=1+3t$ and ordinate, $y=2+2t$

By eliminating t, $\frac{x-1}{3}=\frac{y-2}{2}$ or $2x-2=3y-6$

or $2x-3y+4=0$.

21) Ans: **D)** $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$



Sol:
The coordinates of A are $(a\cos\alpha, a\sin\alpha)$.

The equation of OB is $y=\tan\left(\frac{\pi}{4}+\alpha\right)x$

$\Rightarrow CA\perp OB$

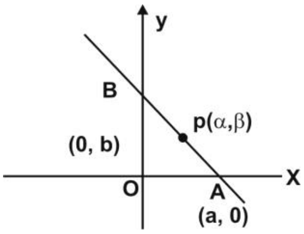
Therefore, slope of $CA=-\cot\left(\frac{\pi}{4}+\alpha\right)$

The equation of CA is $y-a\sin\alpha$

$=-\cot\left(\frac{\pi}{4}+\alpha\right)(x-a\cos\alpha)$

$\Rightarrow y(\sin\alpha+\cos\alpha)+x(\cos\alpha-\sin\alpha)=a$

22) Ans: **B)** $2\alpha\beta$



Sol:

Area of $\triangle OAB = S = \frac{1}{2}ab$ ...(1)

Equation of AB is $\frac{x}{a}+\frac{y}{b}=1$ Putting (α,β),

we get $\frac{\alpha}{a}+\frac{\beta}{b}=1$

$\Rightarrow \frac{\alpha}{a}+\frac{\alpha\beta}{2S}=1$ [using (1)]

$\Rightarrow a^2\beta-2aS+2\alpha S=0 \quad \therefore a\in R \ \Rightarrow D\geq 0$

$\Rightarrow 4S^2-8\alpha\beta S\geq 0 \ \Rightarrow S\geq 2\alpha\beta$.

Therefore, least value of $S=2\alpha\beta$

23) Ans: **A)** $25/16$

Sol: $L\equiv x+y=2$ and $L'\equiv 2x-2y=1$

Equation of y-axis is $x=0$

The vertices of the triangle are $A(0,2), B\left(0,-\frac{1}{2}\right)$ and $C\left(\frac{5}{4},\frac{3}{4}\right)$

$\therefore$ The area of the triangle is $\frac{1}{2}\begin{vmatrix}0 & 2 & 1\\ 0 & -\frac{1}{2} & 1\\ \frac{5}{4} & \frac{3}{4} & 1\end{vmatrix}=\frac{25}{16}$

24) Ans: **B)** $(13/5, 0)$

Sol: Let coordinates of A be (a,0), the slope of reflected ray is $\frac{3-0}{5-a}=\tan\theta$,

Slope of the incident ray $=\frac{2-0}{1-a}=\tan(\pi-\theta)$

$\tan\theta+\tan(\pi-\theta)=0 \quad \Rightarrow \frac{3}{5-a}+\frac{2}{1-a}=0$

$\Rightarrow 13-5a=0 \quad \Rightarrow a=\frac{13}{5}$

$\therefore$ The coordinates of A are $\left(\frac{13}{5},0\right)$

25) Ans: **2** Sol: The lines $ax+by+p=0$ and $x\cos\alpha+y\sin\alpha=p$ are inclined at an angle $\frac{\pi}{4}$

$\Rightarrow \tan\frac{\pi}{4}=\dfrac{-\frac{a}{b}+\frac{\cos\alpha}{\sin\alpha}}{1+\frac{a\cos\alpha}{b\sin\alpha}}$

$\Rightarrow a\cos\alpha+b\sin\alpha=-a\sin\alpha+b\cos\alpha$ .....(i)

The lines $ax+by+p=0$, $x\cos\alpha+y\sin\alpha-p=0$ and $x\sin\alpha-y\cos\alpha=0$ are concurrent.

$\Rightarrow \begin{vmatrix}a & b & p\\ \cos\alpha & \sin\alpha & -p\\ \sin\alpha & -\cos\alpha & 0\end{vmatrix}=0$

$\Rightarrow -ap\cos\alpha-bp\sin\alpha-p=0$

$\Rightarrow -a\cos\alpha-b\sin\alpha=1$

$\Rightarrow a\cos\alpha+b\sin\alpha=-1$ ...(ii)

From (i) and (ii), $-a\sin\alpha+b\cos\alpha=-1$

From (ii) and (iii),

$(a\cos\alpha+b\sin\alpha)^2+(-a\sin\alpha+b\cos\alpha)^2=2$

$\therefore a^2+b^2=2$